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Shock-Layer Radiation for Yawed Cones with Radiative Decay

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APPROXIMATE methods for determination of the effect of radiative decay on the shock-layer radiation toward sphere-cones under zero angle of attack have recently been presented by Chin and Hearne.[1] In this note, these methods are extended for application along the windward line region of circular cones at finite angle of attack.

Figure 1 shows the geometry of a circular cone of half-angle θ_c at an angle of attack α with the freestream. The shock layer is assumed to be very thin compared with the local radius so that the half-angle of the conical shock wave θ_s is approximately that of the body, or $\theta_s \approx \theta_c \approx \theta$. In the immediate region along the windward line, the three velocity components at the entry point (S_i, ϕ_i) immediately behind the shock wave can be shown to be

$$u_c/u_\infty \approx \cos(\theta + \alpha) \tag{1}$$

$$u_\phi/u_\infty \approx \phi \sin\alpha \tag{2}$$

$$u_s/u_\infty \approx \epsilon \sin(\theta + \alpha) \tag{3}$$

where u_c, u_ϕ, and u_s are the velocity components along a conical ray, in the ϕ direction, and normal to the shock wave, respectively; and ϵ is the density ratio across the oblique shock wave.

The exact path of a fluid particle subsequent to its entry within the radiating shock layer cannot be simply determined. However, the entry point (S_i, ϕ_i) for the streamline passing the point (S, ϕ, y_a) may be approximated in several alternative ways. Considered first is the case in which the peripheral acceleration due to the ϕ pressure gradient is neglected. The streamlines follow a geodesic path. The magnitude and direction of the velocity remain unchanged in the developed surface. For this case, the time required for a particle to travel from (S_i, ϕ_i) to (S, ϕ, y_a) may be written as

$$t = (S - S_i)/u_c = [S \sin\theta(\phi - \phi_i)]/[u_\phi(\phi_i)] \tag{4}$$

where $u_\phi(\phi_i)$ is a function of ϕ_i, and

$$y_a = \int_0 \frac{\rho}{\rho_s}\, dy$$

is the Howarth-Dorodnitsyn variable.

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From a continuity consideration,

$$u_s S_i \sin\theta dS_i d\phi_i = u_c S \sin\theta dy_a d\phi \tag{5}$$

By manipulation of Eqs. (1–5), the following equation may be derived:

$$d(y_a/S) = \epsilon \tan(\theta + \alpha)\zeta d\zeta/(1 + C - C\zeta) \tag{6}$$

where

$$\zeta = S_i/S \qquad C = \sin\alpha/[\cos(\theta + \alpha)\sin\theta]$$

The parameter C may be termed a "crossflow" parameter. Integration of Eq. (6) yields the following expression:

$$\frac{y_a}{S} = \frac{2}{C}\left\{\frac{1}{C}(1 + C)\ \ln\left(\frac{1 + C}{1 + C - C\zeta}\right) - \zeta\right\}\frac{\epsilon}{2}\ \tan(\theta + \alpha) \tag{7}$$

By letting $\zeta = 1$, Eq. (7) yields

$$\frac{\Delta a}{S} = F\,\frac{\epsilon}{2}\tan(\theta + \alpha) \tag{8}$$

where Δ_a is the adiabatic shock-layer thickness, and

$$F \equiv \frac{\Delta_a/S}{(\epsilon/2)\tan(\theta + \alpha)} = 2\,\frac{(1 + C)\ln(1 + C) - C}{C^2} \tag{9}$$

When $\alpha = 0$, $F = 1$, and Eq. (8) becomes identical with the constant-density solution for a circular cone at zero angle of attack.[2] The factor F therefore represents the effect of crossflow on Δ_a/S at the windward line relative to the case with an equivalent cone half-angle $(\theta + \alpha)$ and without crossflow; hence, F may be termed the crossflow factor.

Equations (7) and (8) may be combined to yield

$$\eta \equiv \frac{y_a}{\Delta_a} = \frac{(1 + C)\ln\left(\frac{1 + C}{1 + C - C\zeta}\right) - C\zeta}{(1 + C)\ln(1 + C) - C} \tag{10}$$

The energy balance along the streamline may be written as follows:

$$-u(dh/ds) = u_c(dh/dS_i) = 4\sigma k T^4 \tag{11}$$

where u is the velocity along the streamline direction s, h the enthalpy, k the Planck-mean mass absorption coefficient, σ the Stefan-Boltzmann constant, and T the absolute temperature.

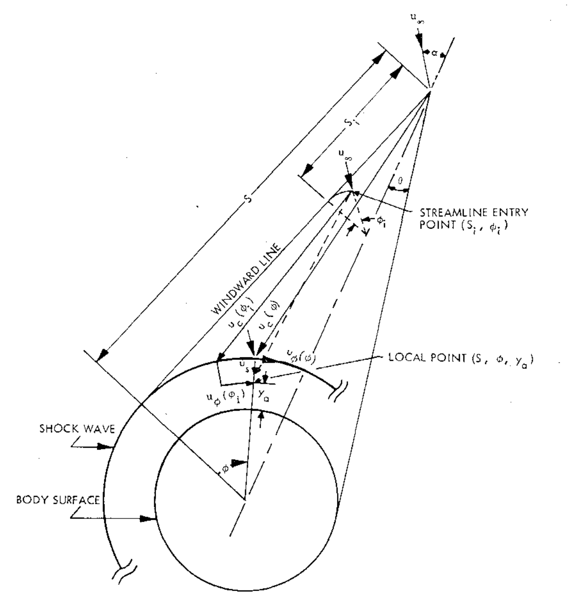

Fig. 1 Geometry of circular cone at angle of attack.

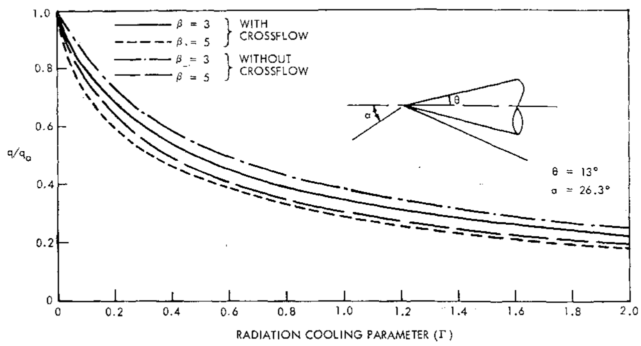

Fig. 2 Effect of radiative decay on radiative flux (example).

In a first-order radiation calculation, the following correlation may be used:

$$(\partial \ln kT^4/\partial \ln h)_p \equiv \beta \approx \text{const} \tag{12}$$

Equation (11) may be integrated with use of Eqs. (1) and (12). By means of Eq. (8), the integrated result may be put in the form

$$h/h_s = [1 + 2(\beta - 1)\Gamma(1 - \zeta)/F]^{-1/(\beta-1)} \tag{13}$$

where

$$\Gamma = 2q_a/\rho_s u_s h_s \qquad h_s \approx \tfrac{1}{2}u_\infty^2 \sin^2(\theta + \alpha)$$

$$q_a = 2\rho_s k_s \sigma T_s^4 \Delta_a$$

which is the equation for adiabatic shock-layer radiation.

The nonadiabatic shock-layer radiation to the vehicle surface is given by

$$q = \int_0^{\Delta_a} 2\rho_s k \sigma T^4 dy_a \tag{14}$$

Equation (14) may be transformed to yield

$$\frac{q}{q_a} = \int_0^1 \frac{kT^4}{k_s T_s^4} d\eta \tag{15}$$

By means of Eqs. (10, 12, and 13), Eq. (15) may be manipulated to yield

$$\frac{q}{q_a} = \frac{2}{F}\int_0^1 (1 + K - K\zeta)^{-\beta/(\beta-1)} \left(\frac{\zeta}{1 + C - C\zeta}\right) d\zeta \tag{16}$$

where $K = [2(\beta - 1)\Gamma]/F$. When $K = C$, the integration of the integral in Eq. (16) is simple, and the result is

$$\frac{q}{q_a} = \frac{2(\beta - 1)}{CF}\left\{\frac{1}{\beta} - \left(\frac{\beta - 1}{C\beta}\right) \times [1 - (1 + C)^{-1/(\beta-1)}]\right\} \qquad (K = C) \tag{17}$$

The integration is more difficult for $K \neq C$. However, the following expressions can be derived for $\beta = 3$ and $\beta = 5$:

$$\frac{q}{q_a} = \frac{4}{CF}\left\{\left(\frac{1 + C}{K - C}\right)\{[1 - (1 + K)^{-1/2}] + g_3[\tan^{-1}g_3 - \tan^{-1}g_3(1 + K)^{1/2}]\} - \frac{1}{K}[1 - (1 + K)^{-1/2}]\right\}$$
$$(K > C,\ \beta = 3) \tag{18a}$$

$$= \frac{4}{CF}\left\{\left(\frac{1 + C}{K - C}\right)\{[1 - (1 + K)^{-1/2}] - g_3[\coth^{-1}g_3 - \coth^{-1}g_3(1 + K)^{1/2}]\} - \frac{1}{K}[1 - (1 + K)^{-1/2}]\right\}$$
$$(K < C,\ \beta = 3) \tag{18b}$$

where $g_3 = |C/(K - C)|^{1/2}$

$$\frac{q}{q_a} = \frac{8}{CF}\left[\left(\frac{1 + C}{K - C}\right)\left\{[1 - (1 + K)^{-1/4}] - \frac{g_5}{2^{3/2}}\{f_1[g_5(1 + K)^{1/4}] - f_1[g_5]\}\right\} - \frac{1}{K}[1 - (1 + K)^{-1/4}]\right]$$
$$(K > C,\ \beta = 5) \tag{19a}$$

$$= \frac{8}{CF}\left[\left(\frac{1 + C}{K - C}\right)\left\{[1 - (1 + K)^{-1/4}] - \frac{g_5}{2}\{f_2[g_5(1 + K)^{1/4}] - f_2[g_5]\}\right\} - \frac{1}{K}[1 - (1 + K)^{-1/4}]\right]$$
$$(K < C,\ \beta = 5) \tag{19b}$$

where

$$g_5 = |C/(K - C)|^{1/4}$$

$$f_1(W) = \tan^{-1}\left(\frac{2^{1/2}W}{1 - W^2}\right) - \tanh^{-1}\left(\frac{2^{1/2}W}{W^2 + 1}\right)$$

$$f_2(W) = \tan^{-1}W - \coth^{-1}W$$

The value of the angle in the second quadrant should be used, if the argument of the $\tan^{-1}$ is negative.

It is of interest to compare these results with that for zero angle of attack. For zero angle of attack, let $C \to 0$; Eq. (10) may be shown to be

$$\eta = \zeta^2 \tag{20}$$

With $\alpha = C = 0$, $F = 1$, and $\zeta = \eta^{1/2}$, Eqs. (13) and (16) become, respectively,

$$h/h_s = [1 + 2(\beta - 1)\Gamma(1 - \eta^{1/2})]^{-1/(\beta-1)} \tag{21}$$

$$\frac{q}{q_a} = \int_0^1 [1 + 2(\beta - 1)\Gamma(1 - \eta^{1/2})]^{-\beta/(\beta-1)} d\eta = \frac{1}{2(\beta - 1)\Gamma^2}\left\{\frac{1}{\beta - 2} + [1 + 2(\beta - 1)\Gamma] - \left(\frac{\beta - 1}{\beta - 2}\right)[1 + 2(\beta - 1)\Gamma]^{-(\beta-2)/(\beta-1)}\right\} \tag{22}$$

This solution for cones at zero angle of attack has been given in Ref. 1.

The finite and zero angle-of-attack solutions are graphically compared in Fig. 2 for an example situation ($\theta = 13°$, $\alpha =$

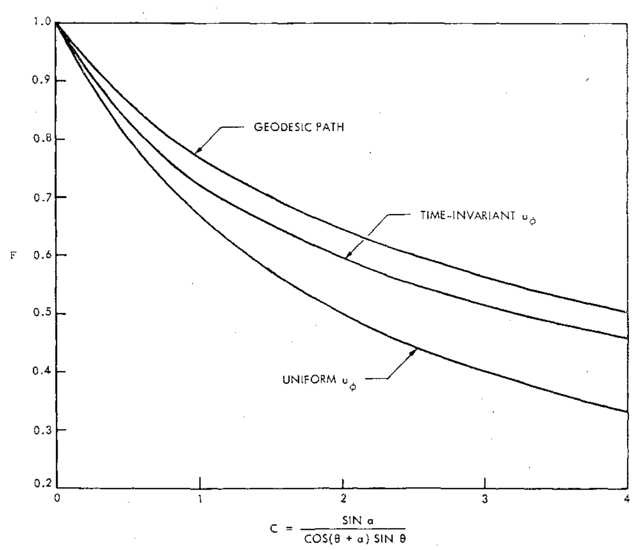

Fig. 3 Crossflow factor for windward line of circular cones at angle of attack.

27°). It is seen that the influence of crossflow on the radiative-flux ratio q/q_a is not large when expressed as a function of the radiation-cooling parameter Γ. Note, however, that the value of Γ is dependent on the crossflow, being linearly related to the crossflow factor F.

The result of Eq. (9) for F may be improved upon by consideration of two alternative approximations for the flow structure. The streamline pattern may be estimated, if it is assumed that the velocity component u_ϕ is time-invariant. In this case, the following equation is used in place of Eq. (4):

$$dt = dS/u_c = (S \sin\theta \, d\phi)/[u_\phi(\phi_i)] \tag{23}$$

With Eq. (23), the expression for F can be shown to be

$$F = (2/C)e^{2/C}[-Ei(-2/C)] \tag{24}$$

where $-Ei(-x)$ is the exponential integral defined by

$$-Ei(-x) = \int_x^\infty \frac{e^{-\xi}}{\xi} d\xi > 0 \qquad \infty > x > 0$$

A final flow approximation is that the velocity component u_ϕ is uniform across the shock layer with $u_\phi = u_\phi(\phi)$. In this case,

$$dt = dS/u_c = (S \sin\theta \, d\phi)/[u_\phi(\phi)] \tag{25}$$

The corresponding expression for F can be shown to be

$$F = 2/(2 + C) \tag{26}$$

The three expressions of F are compared in Fig. 3. The spread of the curves is not large. When the ϕ pressure gradient is taken into account, the F curve is expected to fall between the two curves marked "time-invariant u_ϕ" and "uniform u_ϕ." Therefore, it may be adequate to use the F curve marked "time-invariant u_ϕ" (or Eq. 24) for calculation of the adiabatic shock-layer thickness Δ_a, adiabatic radiation flux q_a, and, hence, the energy loss parameter Γ. As shown by the comparison of Fig. 2, the relatively simple formula of Eq. (22) (or Fig. 2 of Ref. 1) may be used as a good engineering approximation in evaluation of q/q_a.

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Stagnation-Point Heat Transfer and Shock-Detachment Distance for Ellipsoids of Revolution

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Nomenclature

- a = axis of capping ellipse measured perpendicular to body axis
- b = axis of capping ellipse measured along body axis
- h = enthalpy
- h_D = enthalpy of dissociation
- Le = Lewis number
- M_s = Mach number of initial shock wave in shock tube
- P_1 = initial channel pressure in shock tube
- Pr = Prandtl number
- q = stagnation-point heat-transfer rate
- q_h = theoretical stagnation-point heat-transfer rate for a hemisphere
- u_e = velocity component parallel to body surface
- x = distance measured along body surface from stagnation point
- δ = shock-detachment distance
- μ = viscosity
- ρ = density

Subscripts

- s = stagnation conditions
- w = wall conditions

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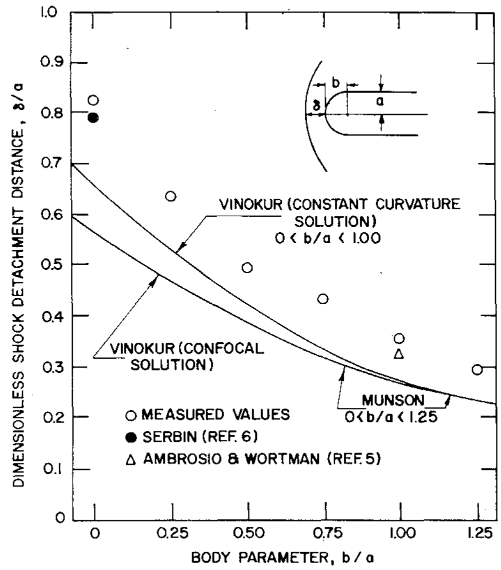

Fig. 1 Dimensionless shock-detachment distance as a function of body parameter, M_s = 5.12, P_1 = 5.25-mm Hg.

Introduction

Considerable stagnation-point, heat-transfer, and shock-detachment data have been reported in the literature for basic shapes, such as the hemisphere cylinder and the flat-faced cylinder. It would seem desirable to have additional data to bridge the gap between these shapes. The purpose of this investigation is to obtain such data for a series of blunt bodies formed by cylindrical afterbodies capped by ellipsoids of revolution. The bodies are characterized by the ratio b/a of the axis of the ellipse along the body axis to the axis perpendicular to the body axis as shown in Fig. 1. The values of the parameter b/a selected for the six models were: 0(flat-faced), 0.25, 0.50, 0.75, 1.00 (hemisphere), and 1.25. The experimentally measured stagnation-point heat-transfer rates and shock-detachment distances were compared with existing theoretical results. The theoretical analysis has been separated into an inviscid analysis of the flow field external to the boundary layer and a viscous boundary-layer analysis.

Boundary-Layer Analysis

A solution for the viscous boundary-layer valid near the stagnation point has been obtained by Fay and Riddell.[1] They wrote the boundary-layer equations including the effects of dissociation and ionization and obtained solutions by numerical methods. Fay and Riddell present the following correlation equation for stagnation-point heat transfer through an equilibrium boundary layer:

$$q = 0.76\, Pr^{-0.6}(\rho_s\mu_s)^{0.4}(\rho_w\mu_w)^{0.1}(h_s - h_w)(du_e/dx)^{0.5} \times [1 + (Le^{0.52} - 1)h_D/h_s] \tag{1}$$

The only term in this equation that depends on the flow field external to the boundary layer is the stagnation-point velocity gradient du_e/dx.